Introduction

The lessons in this section focus on order of operations and on the commutative, associative, distributive, and identity properties.

These lessons form an outline for your ARI classes, but you are expected to add other lessons as needed to address the concepts and provide practice of the skills introduced in the *ARI Curriculum Companion.*

Some of the lessons cross grade levels, as indicated by the SOL numbers shown below. This is one method to help students connect the content from grade to grade and to accelerate.

For the lessons in this section, you will need the materials listed at right.

MATERIALS SUMMARY

Overhead transparencies

Scissors

Balance scale and weights

Counters or chips in various colors (blue, green, yellow)

Two-color counters (red on one side and yellow on the other)

Tape

Grid paper

Standards of Learning

7.2 The student will simplify expressions that contain rational numbers (whole numbers, fractions, and decimals) and positive exponents, using order of operations, mental mathematics, and appropriate tools.

7.3 The student will identify and apply the following properties of operations with real numbers:
- a) the commutative and associative properties for addition and multiplication;
- b) the distributive property;
- c) the additive and multiplicative identity properties;
- d) the additive and multiplicative inverse properties; and
- e) the multiplicative property of zero.

8.1 The student will
- a) simplify numerical expressions involving positive exponents, using rational numbers, order of operations, and properties of operations with real numbers;
- b) recognize, represent, compare, and order numbers expressed in scientific notation; and
- c) compare and order decimals, fractions, percents, and numbers written in scientific notation.

8.4 The student will apply the order of operations to evaluate algebraic expressions for given replacement values of the variables. Problems will be limited to positive exponents.

Table of Contents

The following lessons are included in this section. Click (or CTRL+click) on each to jump to that lesson.

✱ SOL 7.2, 8.1

Prerequisite SOL
6.22, 7.3 (the commutative property)

Lesson Summary
Students discover the order of operations, using the areas of rectangles. (45 minutes)

Materials
Overhead transparency of three rectangles
Copies of the attached worksheet

Warm-up
Review with students the process of finding the area of a rectangle. Use an overhead transparency to display three rectangles with the dimensions (9" x 4", 9" x 5", and 9" x 8") of each rectangle shown. Ask students to explain the most direct way to find the total area of any two of these rectangles. They should recognize that the most direct way is by finding the area of each rectangle and then adding the two areas, e.g., 9 • 4 = 36; 9 • 5 = 45; 36 + 45 = 81. Ask them to explain the most direct way to find the total area of all three rectangles. (Find the area of each, and then add the three areas.)

Lesson
1. Put the following problem on the board: 4 • 3 + 2 • 5 = *x*. Have students solve the problem and then share their answer with a partner. Ask the students whether anyone got an answer of 70. If so, ask how them how they got that answer. They will probably say that they multiplied 4 and 3 to get 12, then added 2 to 12 to get 14, and finally multiplied 14 by 5 to get 70. Ask the students if anyone got an answer of 22. If so, ask how they got that answer. The students who got this answer should explain that they multiplied 4 by 3 to get 12, then multiplied 2 by 5 to get 10, and finally added 12 and 10 to get 22. Discuss with the students why using these two different procedures would give such different answers.
2. Have students write an expression to model the process they used in the warm-up to find the total area of two rectangles. Students should write one of the following expressions: 9 • 4 + 9 • 5, 9 • 4 + 9 • 8, or 9 • 5 + 9 • 8. Select one of these expressions, and use it to model on the board the two procedures used in the warm-up: (1) doing all operations left to right and (2) doing all multiplications first and then doing addition.
3. Have students work in pairs to construct an explanation of why they must use the second procedure when finding the area of two rectangles. Allow plenty of time for discussion.
4. Allow pairs time to present their explanations. At the end, summarize the discussion to explain why the second procedure is correct.
5. Have the students write a rule, based on the discussion, for simplifying expressions that involve multiplication and addition. The rule should state that *all multiplication must be done before any addition.*
6. Have the students write an expression to find the total area of all three rectangles from the warm-up (e.g., 9 • 4 + 9 • 5 + 9 • 8), and have students use their rule to simplify the expression.
7. Have the students work in pairs to see if there is another way to solve their original warm-up problem. Since the rectangles have one dimension the same, they could be put next to each other to create one big rectangle. For example, the 9 x 4 and 9 x 5 rectangles together create a 9 x 9 rectangle with an area of 81. Have students write an expression that models the above approach. One expression could be 9 • (4 + 5). To simplify this expression, explain that 4 and 5 need to be added first and then the sum multiplied by 9 to get 81. Have students write a rule for simplifying expressions that involve parentheses and multiplication. The rule should state that *all expressions inside of parentheses must be simplified before any multiplication.*

8. Ask the students to simplify 3^2 and then add 5 to the answer. Have the students write an expression that models the problem. Students will most likely write $3^2 + 5$. Explain that the commutative property allows the expression also to be written $5 + 3^2$. Have the students write a rule for simplifying expressions that involve exponents and addition. The rule should state that *all exponents must be simplified before any addition.*
9. Have students write a new rule that combines the three rules they have written. The rule should state that *when simplifying, expressions inside parentheses must be simplified first, then all exponents must be simplified, then all multiplication is done, and, finally, all addition is done.* Explain that this "order of operations" rule includes division with multiplication and subtraction with addition. *Multiplication and division must be done from left to right first, and then addition and subtraction is done from left to right.*
10. Help the class create a complete "order of operations" rule similar to the following:

    > *When simplifying, do all expressions inside parentheses first, then all exponents, then all multiplication and division operations from left to right, and finally all addition and subtraction operations from left to right.*

    It might be helpful for students to see how to organize the order of operations in a numbered list of steps, as shown below:

    > 1. *Do any work within parentheses ( ) or other grouping symbols [ ] first.*
    > 2. *Do any work with exponents (powers) or roots.*
    > 3. *Do any multiplication and division in order from left to right.*
    > 4. *Do any addition and subtraction in order from left to right.*

11. Have students create and write two expressions, each containing parentheses, exponents, and all operations. On a separate sheet of paper, have them simplify their expressions, showing each step in the order of operations and the final answer. Have them exchange their problems with a partner, simplify each other's expressions, and discuss the problems until agreement is reached on the correct order of operations and final answer.

Reflection

Have students complete the "Simplifying Expressions" worksheet.

Name: ________________________

Simplifying Expressions

Simplify each expression, showing each step in the order of operations. To the right of each step, identify the step as parentheses, exponents, multiplication, division, addition, or subtraction.

Example

$(4 + 5) \cdot 4 - 3^2 + 9(2)$

$9 \cdot 4 - 3^2 + 9(2)$	parentheses—addition
$9 \cdot 4 - 9 + 9(2)$	exponents
$36 - 9 + 18$	multiplication, left to right
$27 + 18$	subtraction, left to right
45	addition

1. $9 - 2^3$

   ____

   ____

2. $72 - (7 + 8) \cdot 4$

   ____

   ____

   ____

3. $64 - 4 \cdot 2^3 + 7$

   ____

   ____

   ____

   ____

4. $3 + 7(2^3 - 6)^2$

   ____

   ____

   ____

   ____

   ____

Name: ANSWER KEY

Simplifying Expressions

Simplify each expression, showing each step in the order of operations. To the right of each step, identify the step as parentheses, exponents, multiplication, division, addition, or subtraction.

Example

$(4 + 5) \cdot 4 - 3^2 + 9(2)$

$9 \cdot 4 - 3^2 + 9(2)$	parentheses—addition
$9 \cdot 4 - 9 + 9(2)$	exponents
$36 - 9 + 18$	multiplication, left to right
$27 + 18$	subtraction, left to right
45	addition

1. $9 - 2^3$

$9 - 8$	exponents
1	subtraction

2. $72 - (7 + 8) \cdot 4$

$72 - 15 \cdot 4$	parentheses—addition
$72 - 60$	multiplication
12	subtraction

3. $64 - 4 \cdot 2^3 + 7$

$64 - 4 \cdot 8 + 7$	exponents
$64 - 32 + 7$	multiplication
$32 + 7$	subtraction, left to right
39	addition

4. $3 + 7(2^3 - 6)^2$

$3 + 7(8 - 6)^2$	parentheses—exponents
$3 + 7(2)^2$	parentheses—subtraction
$3 + 7(4)$	exponents
$3 + 28$	multiplication
31	addition

SOL 7.2, 8.1

Prerequisite SOL
6.22

Lesson Summary
Students review and practice simplifying expressions, using the order of operations. (45 minutes)

Materials
Copies of the attached worksheets
Scissors

Warm-up
Write "**PEMDAS**" on the board, and explain what each letter stands for—Parentheses, Exponents, Multiplication, Division, Addition, Subtraction. Tell the students that they might want to use a mnemonic device to help remember **PEMDAS** (e.g., "Purple Eggplants Make Delicious Afternoon Snacks"). Let the students make up their own mnemonic devices for **PEMDAS** and share them with the class. Remind students that *multiplication and division must be simplified from left to right before addition and subtraction are simplified from left to right. This should be accounted for in the mnemonic that the students create.*

Lesson
1. Distribute a copy of "Order of Operations 4 x 4 Square" and a pair of scissors to each student. Have students cut the squares apart. (Alternatively, pre-cut the squares and make sets of 16 squares inserted into envelopes for distribution to students.)
2. Have students simplify each expression on a separate piece of paper and record the answer below the printed expression.
3. Have students then match up each answer to one that is already printed on another square. Once all expressions are matched to their correct answers, a new 4 x 4 square will be formed.

Reflection
Hand out a copy of the blank "Order of Operations 3 x 3 Square" worksheet to each student. Have the students create and write an expression on each of two sides of each square (in the shaded boxes), simplify each expression, and write each answer on the matching side of the adjacent square. Have students exchange worksheets with a partner to check and verify their work. (You may wish to use these squares in another class for additional practice.)

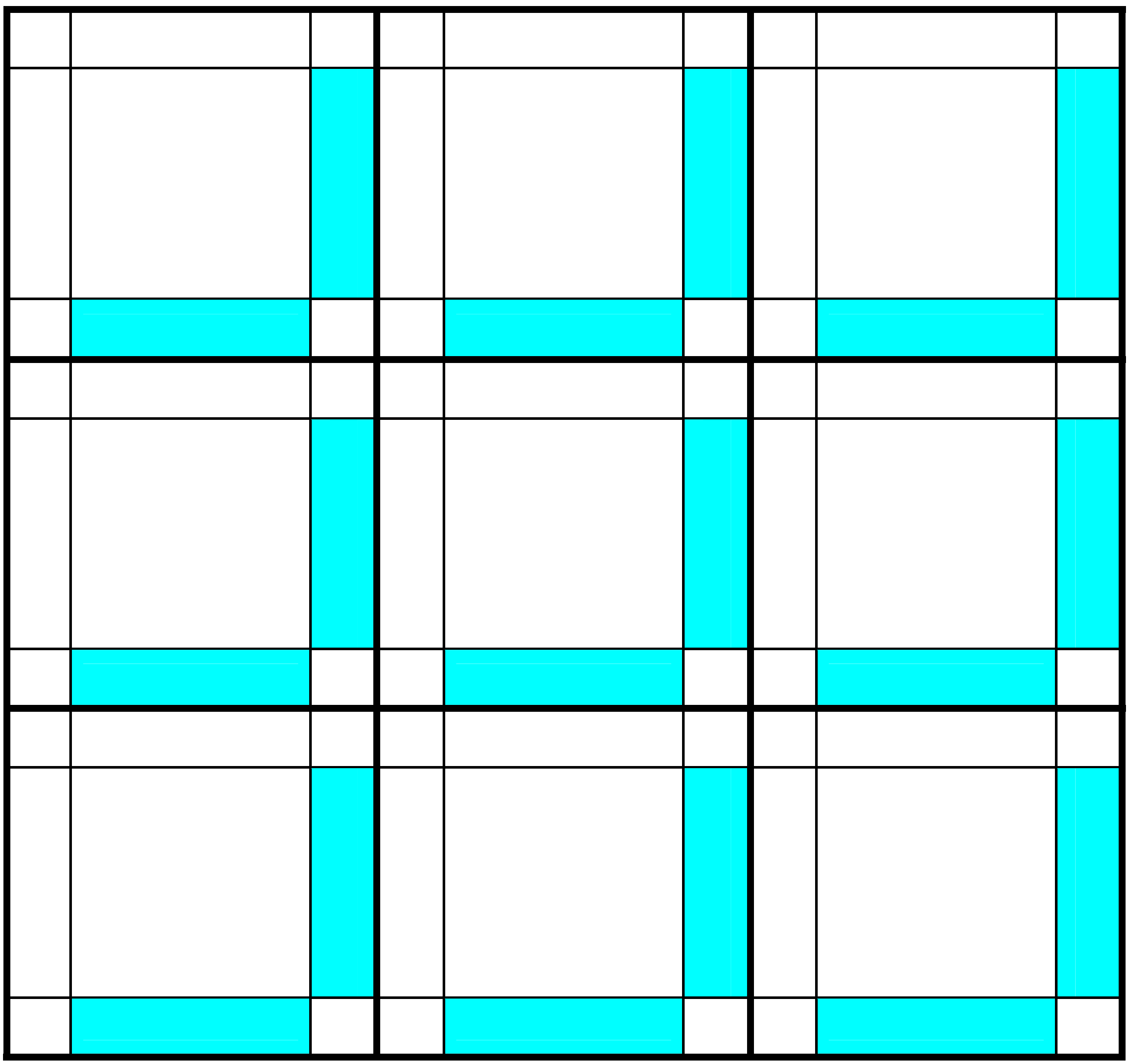
Order of Operations 4 x 4 Square

1. Cut the squares apart along the heavy lines to get 16 small squares.
2. Simplify each expression on a separate sheet of paper, and write each answer below the printed expression.
3. Lay the 16 small squares side by side so that each answer matches up to the same number printed on another square. You should form a new 4 x 4 square.

	24 ÷ 5 + 3			$44 \div 6^2 + 1$			(36 + 4) ÷ 12			$5^2 \div 2 + 4$	
43		16 − 2 • 6 + 1	5		5 • 2 + 4 ÷ 2	12		$(2^2 + 6) \div 5$	2		(5 • 2) + 3
	15			8			11			2	
	$(6 - 2)^2 - 1$			7 + 15 ÷ 3 − 4			21 − 5 • 2			24 ÷ (6 • 2)	
52		25 ÷ (4 + 1)	5		$(2 \cdot 3)^2 - 25$	11		(24 ÷ 6) + (2 • 5)	14		(4 • 7) − 6
	8			16			8			37	
	30 ÷ (1 + 4) + 2			4 • 3 + 8 ÷ 2			24 ÷ 6 • 2			(8 + 4) • (1 + 2) + 1	
28		$(5^2 + 3) \div 2$	14		(6 − 2) • 1 + 6	10		$4 + 6^2 \div 2$	22		$2 \cdot 9^2 \div 13$
	3			36			9			14	
	$6 - (2^2 - 1)$			(30 ÷ 1) + (4 + 2)			8 + 4 ÷ (1 + 2 + 1)			36 ÷ 6 + 2 • 4	
31		14 + 1 − 6 ÷ 3	13		7 − (2 • 6) ÷ 2	1		6 + 5 • 4 − 2	24		17 ÷ 6 + 3
	64			69			72			12	

Order of Operations 3 x 3 Square

1. Create and write an expression on each of two sides of each square (in the shaded boxes).
2. Simplify each expression on a separate sheet of paper, and write each answer on the matching side of the adjacent square.
3. Exchange worksheets with a partner to check and verify your work.
4. Once all your expressions have been checked, cut the 9 small squares apart and re-form the large 3 x 3 square, using the same procedure as was used with the 4 x 4 square.



✱ SOL 7.3

Prerequisite SOL
None

Lesson Summary
Students model and explore the commutative and associative properties of addition and multiplication, using color counters and an equation balance mat. (45 minutes)

Materials
Balance scale and weights
Counters or chips in various colors (blue, green, yellow)
Copies of the attached worksheets
Grid paper

Vocabulary
commutative property of addition. $a + b = b + a$
associative property of addition. $a + (b + c) = (a + b) + c$
commutative property of multiplication. $a \bullet b = b \bullet a$
associative property of multiplication. $a \bullet (b \bullet c) = (a \bullet b) \bullet c$

Warm-up
Ask students to list three ways they could add two positive numbers and get a sum of 4. Possible answers are 1 + 3, 2 + 2, and 3 + 1. Ask the students to explain two ways to solve the problem $5 + 3 + 7 = x$. Possible answers include adding the numbers in order from left to right (5 + 3 = 8 and 8 + 7 = 15) and adding the 3 and 7 first and then adding that sum to the first numbers (3 + 7 = 10 and 10 + 5 = 15). The purpose of this warm-up is to get students thinking that often there can be more than one way to solve a problem.

Lesson
1. Put students into pairs, and explain that they will be exploring some properties of operations. Distribute color counters and a copy of the "Equation Balance Mat" worksheet to each pair of students.
2. Ask students to explain the principle of a balance scale or seesaw. Demonstrate the principle of balance, using a balance scale and weights.
3. Using an equation balance mat on the overhead or the board, model for students the process of balancing an equation, using color counters to represent various quantities. Demonstrate that whatever is changed (added or subtracted) on one side of the mat must be changed equally on the other side for the equation to remain "balanced." Demonstrate the *commutative property of addition* by placing a group of 5 green counters and a group of 3 blue counters on the left side of the equation mat. On the right side of the mat, place a group of 3 blue counters first and then a group of 5 green counters. Write an equation that is represented by this model: 5 + 3 = 3 + 5. Ask students whether the equation is balanced. If necessary, demonstrate a few more examples of the commutative property of addition. Have one student in each pair create an example of the commutative property of addition, using the color counters and the equation balance mat, and have the partner write the equation represented by the model.
4. Demonstrate the *associative property of addition* by placing 3 green counters, 2 blue counters, and 1 yellow counter on the left side of the mat. "Associate" two of these groups together—for example, the green and the blue—by putting one group on top of the other or putting a ring of yarn around them. Ask students how many counters are in this associated group, and then have them add the 1 yellow counter to the group. On the right side of the equation balance mat, place equivalent counters, but this time associate the 2 blue counters with the 1 yellow counter. Ask students for the total number of counters in this associated group, and then have them add the 3 green counters. Explain that this is

an example of the associative property of addition. Have one student in each pair create an example of the associative property of addition, using the color counters and the equation balance mat, and have the partner write the equation represented by this model.

5. Brainstorm with students how they could model the *commutative property of multiplication.* One possibility would be to draw grid-paper arrays that use the areas of several rectangles—for example, a 3 x 4 rectangle, which has an area of 12 units, and a 4 x 3 rectangle, which also has an area of 12 units. Demonstrate this model on an overhead, using a transparency of grid paper. Then, have one student in each pair create an example of the commutative property of multiplication, and have the partner write the equation represented by this model.

Reflection

Have students complete the "Exit Slip" worksheet.

Equation Balance Mat

=

Name: ______________________

Exit Slip

Write an equation to illustrate each of the following properties:

commutative property of addition ______________________________

associative property of addition ______________________________

commutative property of multiplication ______________________________

✱ SOL 7.3

Prerequisite SOL

None

Lesson Summary

Students model and discover the additive identity property and additive inverse property, using counters. They also investigate the multiplicative identity property, the multiplicative inverse property, and the multiplicative property of zero. (30 minutes)

Materials

Two-color counters (red on one side and yellow on the other)

Vocabulary

additive identity property. $a + 0 = a$
multiplicative identity property. $a \cdot 1 = a$
additive inverse property. $a + (-a) = 0$
multiplicative inverse property. $a \cdot \frac{1}{a} = 1$
multiplicative property of zero. $a \cdot 0 = 0$

Warm-up

Hand out a set of two-color counters to each student, and explain that the red side represents positive one (+1) and the yellow side represents negative one (−1). Have each student place 5 positive counters on his/her desk. Ask how many counters could be added to this group to keep the number of counters the same. (none, or 0) Explain that this is an example of the *additive identity property:* $a + 0 = a$. Then, ask students by what number these 5 counters could be multiplied in order to keep the number of counters the same. (1) Explain that this is an example of *multiplicative identity property:* $a \cdot 1 = a$.

Lesson

1. Have students write an equation to model the first warm-up demonstration. (Make sure they note that the equation can be $5 + 0 = 5$ or $0 + 5 = 5$ because of the commutative property of addition.) Explain that the properties that are referred to as "identity properties" are really just common sense. These identity elements are numbers that when combined with other numbers, do not change the original numbers. When 0 is added to any number, the number remains the same. Have students write three equations that demonstrate the *additive identity property,* e.g., $3 + 0 = 3$, $-2 + 0 = -2$, and $\frac{1}{4} + 0 = \frac{1}{4}$.
2. Have students write an equation to model the second warm-up demonstration. (Make sure students note that the equation can be $5 \cdot 1 = 5$ or $1 \cdot 5 = 5$ because of the commutative property of multiplication.) The identity element for multiplication is 1: i.e., when any number is multiplied by 1, the number remains the same. Have students write three equations that demonstrate the *multiplicative identity property,* e.g., $5 \cdot 1 = 5$, $137 \cdot 1 = 137$, and $\frac{3}{8} \cdot 1 = \frac{3}{8}$.
3. Tell students that *inverses* are numbers that combine with other numbers and result in identity elements. The identity element for addition is 0; the identity element for multiplication is 1. For the *additive inverse property,* the sum of the two numbers is zero. Review with students the concepts of negative and positive integers, reminding them of activities using two-color counters—red on one side to represent a positive value, and yellow on the other to represent a negative value. Then, use two-color counters and the equation balance mat (see p. 11) to demonstrate the process of finding the additive inverse. Place 5 red counters on the left side of the equation mat and no counters (representing zero) on the right side of the mat. Ask students what needs to be added to the left side of the mat to have the total value of counters equal zero. Reminding students that the yellow counter represents a negative value of 1, place 5 yellow counters (−5) on the left side of the mat with the 5

red counters (+5). Demonstrate how to make the zero pairs, leaving zero counters on the left side of the mat. Record the process with the equation $5 + (-5) = 0$. Have students write three equations that demonstrate the additive inverse property, e.g., $4 + (-4) = 0$, $-12 + 12 = 0$, $\frac{1}{3} + (-\frac{1}{3}) = 0$.

4. To demonstrate the *multiplicative inverse property,* have students brainstorm which numbers could be used as factors with other numbers to produce a product of 1. Have them use variables in a few examples, such as $4 \cdot x = 1$ or $\frac{1}{3} \cdot x = 1$, to generate ideas about how to find the multiplicative inverse of a number, which is also called the *reciprocal* of the number. Have students write three equations that demonstrate the multiplicative inverse property, e.g., $7 \cdot \frac{1}{7} = 1$, $\frac{1}{2} \cdot 2 = 1$, $\frac{3}{5} \cdot \frac{5}{3} = 1$.
5. For the *multiplicative property of zero,* explain to students that this property is very simple: any number multiplied by zero is equal to zero. Have students write three equations that demonstrate the multiplicative property of zero, e.g., $7 \cdot 0 = 0$, $-3 \cdot 0 = 0$, $\frac{2}{3} \cdot 0 = 0$. Remind students that division by zero is an impossible arithmetic operation.

Reflection

Have students write a paragraph describing the similarities and differences between the additive inverse property and multiplicative inverse property. (Sample answer: Both inverse properties involve finding two numbers that when added or multiplied give the identity of the operation. For addition, the inverse (or *opposite*) of a number is formed by simply switching the sign of the number from positive to negative or vice versa. For multiplication, the inverse (or *reciprocal*) of a number is formed by first representing the number as a fraction, and then switching the numerator and denominator.)

Have students complete the “Exit Slip” worksheet.

Name: ______________________

Exit Slip

Write an equation to illustrate each of the following properties:

additive identity property ______________________________

multiplicative identity property ______________________________

additive inverse property ______________________________

multiplicative inverse property ______________________________

multiplicative property of zero ______________________________

✱ SOL 7.3

Prerequisite SOL

None

Lesson Summary

Students model and discover the distributive property, using grid paper. (30 minutes)

Materials

Attached display sheet showing two grids
Copies of the attached grid sheet
Copies of the attached worksheet
Scissors

Vocabulary

distributive property. $a \cdot (b + c) = a \cdot b + a \cdot c$

Warm-up

Using the overhead, display the attached sheet showing a 3 x 9 grid and a 5 x 9 grid. Ask students how they can determine the area of each grid. (Multiply the width by the height: $3 \cdot 9 = 27$; $5 \cdot 9 = 45$) Next, cut out the grids, and display them placed side by side, touching. Ask students how they can determine the area of this larger, combined grid. After students have had time to respond, point out that there are two ways of determining this area:

- Add the areas of the two, uncombined grids: $(3 \cdot 9) + (5 \cdot 9) = 27 + 45 = 72$
- Multiply the total width by the height: $9 \cdot (3 + 5) = 9 \cdot 8 = 72$.

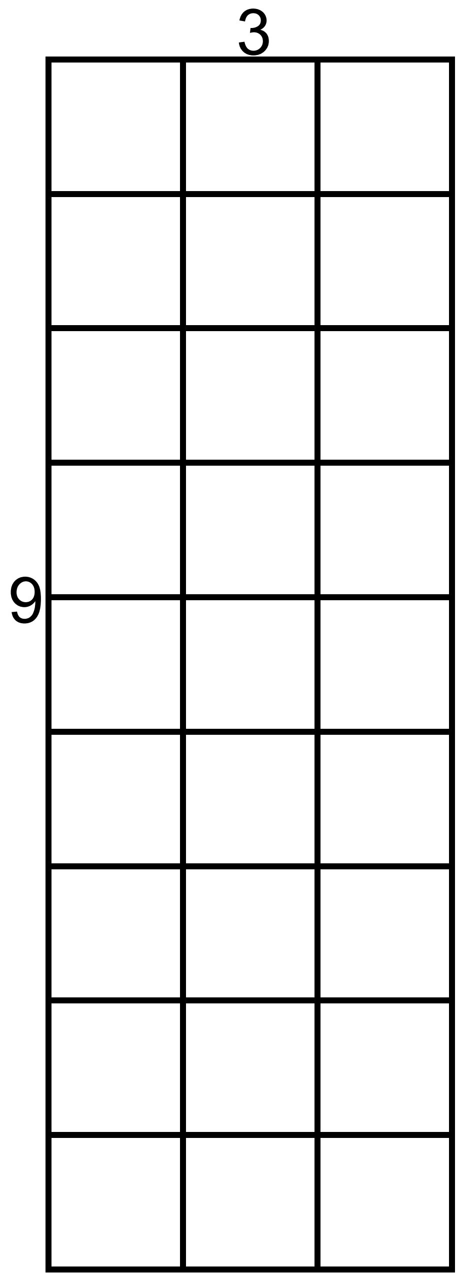
Lesson

1. Explain that the method of finding the area of the two smaller grids and then adding them together is an example of the *distributive property.* Explain that the distributive property lets you "spread out" numbers so they are easier to work with.
2. Write this problem on the board: $4 \cdot 23 = x$. Tell the students that to solve this problem, we can simply multiply 23 by 4, of course. However, there is another way to do this that could be helpful if we must do the calculation in our heads. The distributive property allows us to write this problem as follows: $4 \cdot (a + b) = 4 \cdot a + 4 \cdot b$. In other words, we can "spread out" 23 into the sum of two smaller numbers, for example, 20 + 3. Then, we can do the multiplication operation on each of the smaller numbers and add the two products.
3. Give each student a sheet of grid paper, and have students outline two rectangles on the grid: one of the rectangles should be 20 x 4, and the other should be 3 x 4. Have students write the area of each rectangle inside it and then cut out the two rectangles. Have the students place the rectangles side by side, touching, so that the combined width is 23.
4. Explain again that "spreading out" 23 into 20 + 3 is an example of using the distributive property. Ask students whether there are other ways to spread out 23.
5. Have students model the following problems on grid paper, using the distributive property:
   $5 \cdot 18 = x$ [Possible model: $5 \cdot 18 = 5 \cdot (10 + 8) = 5 \cdot 10 + 5 \cdot 8 = 50 + 40 = 90$]
   $7 \cdot 24 = x$ [Possible model: $7 \cdot 24 = 7 \cdot (20 + 4) = 7 \cdot 20 + 7 \cdot 4 = 140 + 28 = 168$]
   $6 \cdot 27 = x$ [Possible model: $6 \cdot 27 = 6 \cdot (25 + 2) = 6 \cdot 25 + 6 \cdot 2 = 150 + 12 = 162$]
   Have students share and discuss their models with a partner.

Reflection

Have students describe in writing two different ways that they could solve the problem $9 \cdot 57 = x$. Encourage them to include drawings.

Have students complete the "Exit Slip" worksheet.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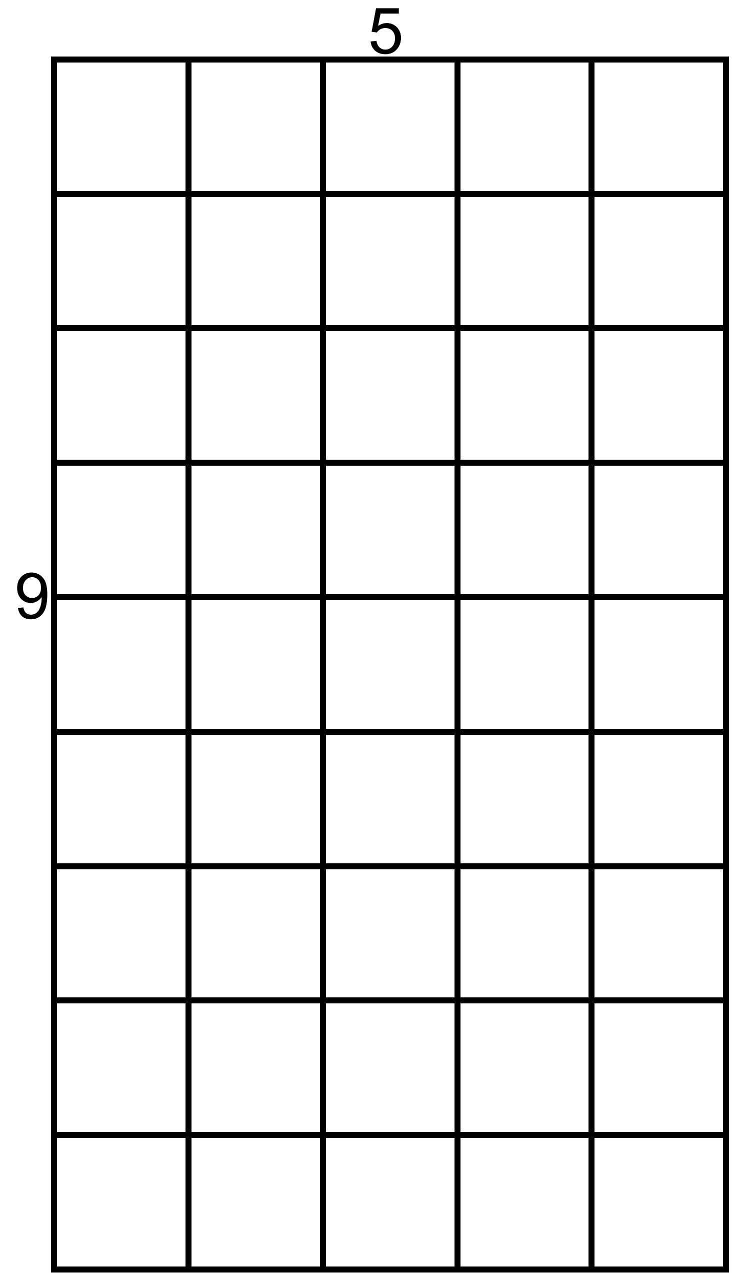
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Name: ________________________

Exit Slip

Write an equation to illustrate the following property:

distributive property ______________________________

✱ SOL 7.3

Prerequisite SOL
None

Lesson Summary
Students review and apply all the properties of operations with real numbers. (30 minutes)

Materials
Copies of the attached worksheets
Sets of attached Property Cards

Vocabulary
commutative property of addition. $a + b = b + a$
commutative property of multiplication. $a \cdot b = b \cdot a$
associative property of addition. $a + (b + c) = (a + b) + c$
associative property of multiplication. $a \cdot (b \cdot c) = (a \cdot b) \cdot c$
distributive property. $a \cdot (b + c) = a \cdot b + a \cdot c$; $a \cdot (b - c) = a \cdot b - a \cdot c$
additive identity property. $a + 0 = a$
multiplicative identity property. $a \cdot 1 = a$
additive inverse property. $a + (-a) = 0$
multiplicative inverse property. $a \cdot \frac{1}{a} = 1$
multiplicative property of zero. $a \cdot 0 = 0$

Warm-up
Distribute copies of the "Warm-up" worksheet, and allow students ample time to complete it. Provide assistance as needed. Have students share their answers to the problems. Discuss the examples of the properties, and answer questions as needed.

Lesson
1. Give each pair of students a set of Property Cards. Have the students shuffle the cards and place them face down in a 4 x 5 arrangement.
1. Explain the Property Card Game, as follows: On each turn, one student turns over two cards. If the cards are a match, the student keeps the cards and continues playing until he or she turns over two cards that do not match. The next student then takes a turn. The winner is the student who has the most cards after all cards are taken.
2. Have pairs of students play the game several times.

Reflection
Have students complete the "Exit Slip" worksheet.

Name: ________________________

Warm-up

For each property of operations listed below, write on the line next to it one of the given equations that demonstrates it.

Equations

$2(-5 + 3) = 2(-5) + 2(3)$	$11 + (-2) = (-2) + 11$
$11 + 0 = 11$	$\frac{1}{3} \cdot 3 = 1$
$4 \cdot (-2) = -2 \cdot 4$	$-7 + (1 + 8) = (-7 + 1) + 8$
$3 \cdot 0 = 0$	$-15 \cdot 1 = -15$
$-5 \cdot (7 \cdot 4) = (-5 \cdot 7) \cdot 4$	$9 + (-9) = 0$

1. commutative property of addition ________________________
2. commutative property of multiplication ________________________
3. associative property of addition ________________________
4. associative property of multiplication ________________________
5. distributive property ________________________
6. additive identity property ________________________
7. multiplicative identity property ________________________
8. additive inverse property ________________________
9. multiplicative inverse property ________________________
10. multiplicative property of zero ________________________

Name: ANSWER KEY

Warm-up

For each property of operations listed below, write on the line next to it one of the given equations that demonstrates it.

Equations

$2(-5 + 3) = 2(-5) + 2(3)$	$11 + (-2) = (-2) + 11$
$11 + 0 = 11$	$\frac{1}{3} \cdot 3 = 1$
$4 \cdot (-2) = -2 \cdot 4$	$-7 + (1 + 8) = (-7 + 1) + 8$
$3 \cdot 0 = 0$	$-15 \cdot 1 = -15$
$-5 \cdot (7 \cdot 4) = (-5 \cdot 7) \cdot 4$	$9 + (-9) = 0$

1. commutative property of addition $11 + (-2) = (-2) + 11$
2. commutative property of multiplication $4 \cdot (-2) = -2 \cdot 4$
3. associative property of addition $-7 + (1 + 8) = (-7 + 1) + 8$
4. associative property of multiplication $-5 \cdot (7 \cdot 4) = (-5 \cdot 7) \cdot 4$
5. distributive property $2(-5 + 3) = 2(-5) + 2(3)$
6. additive identity property $11 + 0 = 11$
7. multiplicative identity property $-15 \cdot 1 = -15$
8. additive inverse property $9 + (-9) = 0$
9. multiplicative inverse property $\frac{1}{3} \cdot 3 = 1$
10. multiplicative property of zero $3 \cdot 0 = 0$

Property Cards

$1 + 5 = 5 + 1$	**commutative property of addition**
$7 + [3 + (-4)] = (7 + 3) + (-4)$	**associative property of addition**
$8 \bullet 11 = 11 \bullet 8$	**commutative property of multiplication**
$-2 \bullet (5 \bullet 9) = (-2 \bullet 5) \bullet 9$	**associative property of multiplication**
$-4(2 + 7) = -4(2) + -4(7)$	**distributive property**

$12 + 0 = 12$	**additive identity property**
$-17 \cdot 1 = -17$	**multiplicative identity property**
$6 + (-6) = 0$	**additive inverse property**
$8 \cdot \frac{1}{8} = 1$	**multiplicative inverse property**
$6 \cdot 0 = 0$	**multiplicative property of zero**

Name: ______________________

Exit Slip

Circle the correct answer to each question. Beside each *incorrect* answer, write a statement explaining why it is incorrect.

1. **Which is equivalent to the following?**

$$7 \cdot 3 + 4 \cdot 6$$

A $7 \cdot 3 + 6 \cdot 4$
B $7 \cdot 4 + 3 \cdot 6$
C $25 \cdot 6$
D $7 \cdot 42$

2. **Which is an example of the associative property of multiplication?**

F $7 \cdot 0 \cdot 9 = 0$

G $4 \cdot (7 \cdot {}^{-}3) = 4 \cdot ({}^{-}3 \cdot 7)$

H $\left(6 \cdot \frac{1}{6}\right) \cdot 3 = 3$

J $5 \cdot (3 \cdot {}^{-}8) = (5 \cdot 3) \cdot {}^{-}8$

3. Complete these sentences:

My favorite property of operations is ______________________________ because

____.

My least favorite property of operations is ______________________________ because

____.

Name: ANSWER KEY

Exit Slip

Circle the correct answer to each question. Beside each *incorrect* answer, write a statement explaining why it is incorrect.

1. **Which is equivalent to the following?**

$$7 \cdot 3 + 4 \cdot 6$$

A $7 \cdot 3 + 6 \cdot 4$

B $7 \cdot 4 + 3 \cdot 6$

C $25 \cdot 6$

D $7 \cdot 42$

B) The 3 and 4 cannot be switched because each is part of a product.

C) The 7 and 3 were multiplied, and then 4 was added to the product. This is not the correct order of operations.

D) The 3 and 4 were added first, and then the sum was multiplied by 6. This is not the correct order of operations.

2. **Which is an example of the associative property of multiplication?**

F $7 \cdot 0 \cdot 9 = 0$

F) This is an example of the multiplicative property of zero.

G $4 \cdot (7 \cdot {}^{-}3) = 4 \cdot ({}^{-}3 \cdot 7)$

G) This is an example of the commutative property.

H $\left(6 \cdot \frac{1}{6}\right) \cdot 3 = 3$

H) This is an example of the multiplicative inverse property as well as the multiplicative identity property.

J $5 \cdot (3 \cdot {}^{-}8) = (5 \cdot 3) \cdot {}^{-}8$

✱ SOL 8.4

Prerequisite SOL
7.2, 8.1

Lesson Summary
Students evaluate algebraic expressions, using physical representations of numbers as replacement values of variables. (45 minutes)

Materials
Copies of attached worksheets
Attached number sheets
Tape

Warm-up
Have students complete the “Warm-up” worksheet. Once they have completed the task, review the solutions, and answer any questions they may have.

Lesson
1. Cut apart the attached number sheets so that each large-sized number is on one sheet.
2. Write the expression $2(3 + x) - 8$ on the board. Tell students that the variable, x, in this expression can have many different values and that you are going to replace the x with a number. Tape one of the large numbers over the x, and evaluate the expression on the board.
3. Have students choose several different values for x from the number sheets. Tape each chosen value over the x, and have the students evaluate the resulting expressions. Then, show the evaluation of each expression on the board so that students can confirm or correct their own evaluation.
4. Write the expression $4a - 2b \cdot 3$ on the board. Have two students come up to the board, and have one choose a value for a from the number sheets and the other choose a value for b. Have the students tape their chosen numbers in the appropriate places over a and b. Explain that the substituted numbers must be enclosed in parentheses in order for the side-by-side digits to indicate multiplication. For example, if 3 is substituted for a, 4(3) would look like 43 without the parentheses; if −5 is substituted for a, 4(−5) would look like 4−5 without the parentheses.
5. Have the two students at the board work together to evaluate the expression they created.
6. Have other pairs of students come up to the board to choose replacement values for a and b, tape the numbers over the variables, and evaluate the expression on the board while the other students evaluate the expression on paper.
7. Have the students complete the “Evaluating Expressions Practice” worksheet. Provide assistance as needed.

Reflection
Have students complete the “Exit Slip” worksheet.

Name: ________________________

Warm-up

Evaluate each expression, showing each step in the order of operations.

1. $5 + 5 \cdot 3 - 2 \cdot 2^2$

   ____

   ____

   ____

   ____

2. $4 \cdot 7 + 8 \div 2^3$

   ____

   ____

   ____

   ____

3. $5^2 \div 5 \cdot (2 + 4)$

   ____

   ____

   ____

   ____

4. $(12 - 8)^2 - 4 + 21$

   ____

   ____

   ____

   ____

5. $7 - 1 \cdot 2(2^4 \div 8)$

   ____

   ____

   ____

   ____

   ____

Name: ANSWER KEY

Warm-up

Evaluate each expression, showing each step in the order of operations.

1. $5 + 5 \cdot 3 - 2 \cdot 2^2$

   $5 + 5 \cdot 3 - 2 \cdot 4$

   $5 + 15 - 8$

   $20 - 8$

   12

2. $4 \cdot 7 + 8 \div 2^3$

   $4 \cdot 7 + 8 \div 8$

   $28 + 8 \div 8$

   $28 + 1$

   29

3. $5^2 \div 5 \cdot (2 + 4)$

   $5^2 \div 5 \cdot 6$

   $25 \div 5 \cdot 6$

   $5 \cdot 6$

   30

4. $(12 - 8)^2 - 4 + 21$

   $(4)^2 - 4 + 21$

   $16 - 4 + 21$

   $12 + 21$

   33

5. $7 - 1 \cdot 2(2^4 \div 8)$

   $7 - 1 \cdot 2(16 \div 8)$

   $7 - 1 \cdot 2(2)$

   $7 - 2(2)$

   $7 - 4$

   3

1

2

3

4

5

6

7

8

9

0

−1

−2

−3 −4

−5

−6

−7

−8

Name: ______________________

Evaluating Expressions Practice

Evaluate each expression, showing each step in the order of operations in the spaces to the right of each problem.

1. What is the value of $3z + 2(z - 1)$ when $z = 5$?

2. What is the value of $4 - 2b^2 + 3c$ when $b = 2$ and $c = -1$?

3. What is the value of $6 + x \div 3 \bullet y$ when $x = 15$ and $y = 2$?

4. What is the value of $4(-1 + a)^5 - 3a^2$ when $a = 3$?

5. What is the value of $4m + n - p^3$ when $m = -2$, $n = 5$, and $p = 4$?

Name: ANSWER KEY

Evaluating Expressions Practice

Evaluate each expression, showing each step in the order of operations in the spaces to the right of each problem.

1. What is the value of $3z + 2(z - 1)$ when $z = 5$?	$3(5) + 2(5 - 1)$ $3(5) + 2(4)$ $15 + 8$ 23
2. What is the value of $4 - 2b^2 + 3c$ when $b = 2$ and $c = -1$?	$4 - 2(2)^2 + 3(-1)$ $4 - 2(4) + 3(-1)$ $4 - 8 + (-3)$ $-4 + (-3)$ -7
3. What is the value of $6 + x \div 3 \bullet y$ when $x = 15$ and $y = 2$?	$6 + 15 \div 3 \bullet 2$ $6 + 5 \bullet 2$ $6 + 10$ 16
4. What is the value of $4(-1 + a)^5 - 3a^2$ when $a = 3$?	$4(-1 + 3)^5 - 3(3)^2$ $4(2)^5 - 3(3)^2$ $4(32) - 3(9)$ $128 - 27$ 101
5. What is the value of $4m + n - p^3$ when $m = -2$, $n = 5$, and $p = 4$?	$4(-2) + 5 - (4)^3$ $4(-2) + 5 - 64$ $-8 + 5 - 64$ $-3 - 64$ -67

Name: ________________________

Exit Slip

Evaluate each expression, showing each step in the order of operations in the spaces to the right of each problem. Circle each correct answer.

1. **What is the value of $n^2(m + r)$ if $m = 3$, $n = 2$, and $r = 4$?**

   A 28
   B 16
   C 14
   D 9

2. **What is the value of $x^2(7 - x) + 2$ when $x = 5$?**

   F 52
   G 100
   H 152
   J 172

3. **What is the value of $x - (3x + 5)$ when $x = {}^-2$?**

   A $^-1$
   B 1
   C 5
   D 9

4. **What is the value of $6n(n - 1) + 4$, when $n = 3$?**

   F 44
   G 40
   H 36
   J 19

Name: <u>ANSWER KEY</u>

Exit Slip

Evaluate each expression, showing each step in the order of operations in the spaces to the right of each problem. Circle each correct answer.

1. **What is the value of $n^2(m + r)$ if $m = 3$, $n = 2$, and $r = 4$?**

   A 28
   B 16
   C 14
   D 9

   $2^2(3 + 4)$
   $2^2(7)$
   $4(7)$
   28

2. **What is the value of $x^2(7 - x) + 2$ when $x = 5$?**

   F 52
   G 100
   H 152
   J 172

   $5^2(7 - 5) + 2$
   $5^2(2) + 2$
   $25(2) + 2$
   $50 + 2$
   52

3. **What is the value of $x - (3x + 5)$ when $x = {}^-2$?**

   A $^-1$
   B 1
   C 5
   D 9

   $-2 - [3(-2) + 5]$
   $-2 - (-6 + 5)$
   $-2 - (-1)$
   $-2 + 1$
   -1

4. **What is the value of $6n(n - 1) + 4$ when $n = 3$?**

   F 44
   G 40
   H 36
   J 19

   $6(3)(3 - 1) + 4$
   $6(3)(2) + 4$
   $18(2) + 4$
   $36 + 4$
   40